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How well are the Elders?

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1. Abstract

Increasing trend of elderly population with significant multimorbidity, impairment, disability and psycho-social-economic problems leading to poor quality of life (QOL) is a major public concerns at global level. Methodologically sound measurement of QOL for elderly is the first step to know how well our elders are. Scales to measure QOL and associated factors suffer from methodological limitations and are not comparable due to different dimensions covered; different item formats, scoring methods, method of aggregation, etc. The paper transforms item scores to normally distribute P_i-scores, and scale score $S = \sum_i P_i$ reflecting overall QOL-status (QOL_{Total}) following normal distribution. Another QOL index of elderly persons at *t*-th time period ($I_{QOL-elderly_t}$) is also proposed by multiplicative aggregation of ratios of S_i's at *t*-th period and base period as $I_{QOL-elderly_t} = \frac{S_{1t}.S_{2t}..........S_{nt}}{S_{10}.S_{20}......S_{n0}} * 100$. Highly correlated QOL_{Total} and $I_{QOL-elderly_t}$ satisfy desired properties including measurement of a country/region and plotting progress path with respect to fixed or varying base period. The proposed measures are improvement of QOL for elderly persons with benefits of integration of scales and parametric analysis across time and space. Improved measures of reliability and validity also help in comparison of multidimensional QOL scales.

2. Keywords

Composite index, Equivalent scores, Factorial validity, Multiple regression, Normal distribution, Quality of life, Theoretical reliability

3. Introduction

Increasing size of elderly population (> 60 years of age) with significant multimorbidity rates, impairments, disability and psycho-social problems leading to poor quality of life (QOL) is a major public health concerns at global level. Increasing trend of economic burden on the working age population (WAP) reflected by old-age dependency ratio (DR_{Oldage}) defined as number of old-age dependents per 100 WAP adds to the challenges of a nation for providing effective community-level measures [1]. DR_{Oldage} for females > DR_{Oldage} for males and rate of increase due to higher longevity of women leading to higher elderly-female population. Nations have taken up agenda of active ageing towards enhancement of capabilities, rights, and resilience among the elderly people to achieve the 2030 Agenda of Sustainable Development Goals (SDG) so that none is left behind.

Attainment of retirement age forces persons to withdraw from working environment and enter into the elderly population with loss of socio-economic role, reduced social network, impairment of sensory functions including memory function, dependence on others, managing money, paying bills, etc. resulting in feelings of disorientation, loneliness, lack of self-belief, lost identity, fear of dying, etc. [2,3]. Similar effects are also resulted by widowhood, creating hindrance to a better QOL. Poor schooling, poor social security, low vision, impaired marital status, family structure, etc. affect adversely QOL among elderly population [4].

Like any stage of life, retirement has its highs and lows. One can find his/her new passion; decide how best to live the next phase of life by doing something challenging and contributing to the society and thus, discover the new-self with new identity. However, as senior citizen, retirees may take up different roles and need to adjust with changed self-concept, meaning in life, and also sources from which meaning is derived. They may find it difficult to move forward with new passion and new identity with declining physical and mental health, social relationships, emotional disturbances, and financial constraints. Change in terms of less availability of autonomy, money, lost identities, higher dependency on others due to sensory and cognitive impairments, socio-economic vulnerabilities, discriminations based on age, loss of marital partner(widow/widower), etc. may even lead to compromising dignity and human rights [5] and affect QOL of elders differently in short-term, mid-term and long-term periods [3,6]. Healthy-aging defined by WHO (2022) (www.who.int/news-room/questions-andanswers/) as the process of developing and maintaining functional ability enabling wellbeing for elderly is not ensured in post-retirement life.

Methodologically sound measurement of QOL for elderly is the first step to know how well our elders are. This helps planners to decide or redefine public policies considering among others gradual shift towards older populations. However, selection of dimensions and indicators depends on the purpose. For example, QOL to increase life expectancy of older people are different from the purpose of providing healthier lives with increased opportunities and lower costs to older persons, their families and society. Common indicators could be percentage of older persons along with socioeconomic implications of elderly population like DR_{Oldage}, remaining life expectancy, etc. Generic scales like World Health Organization Quality of Life Abbreviated Version (WHOQOL-BREF), Short Form Health Survey questionnaire (SF-36) and shorter version (SF-12) are popular to assess QOL for elderly. However, SF-12 ignores the important health variable called "sleep". WHOQOL-BREF covers dimensions like physical and psychological health, social relationships and environment with 26 number of 5-point items where number of items and indicators vary across the dimensions. Dimension scores are computed using summative score of items belonging to a dimension ensuring higher score $\Rightarrow$ better QOL for that dimension. But, overall WHOQOL-BREF score is not computed for an individual. SF-36 has seven binary items, 3-point items (10 numbers), 5-point items (8 numbers), 6-point items (10 numbers) and another item regarding health transition over the last year. Mean, SD and distribution are different for Yes-No type, 3-point, 5-point, 6-point items. The manual of SF-36 (http://www.webcitation.org/6cfeefPkf) does not allow total score of an individual since several independent dimensions are measured by the scale. Negative correlation of SF-36 with Patient Health Questionnaire (PHQ) and General Anxiety Disorder (GAD-7) were primarily due to non-uniform factor structures [7]. Because of methodological deficiencies in most Randomized controlled trials (RCTs), arbitrary selection of QOL tools, several suggestions were made including replacing SF-12 by PHQ-9or (GAD-7) [8]. QOL-index for elderly by Institute for Competitiveness, India (https://www.competitiveness.in) considers secondary data on 45 indicators distributed over 8 sub-pillars and 4-pillars: Financial Well-being, Social Well-being, Health System and Income Security of Indians in age-category > 60 and computes QOL-index = $\frac{\sum_{i=1}^{4} Pillar\ score_i}{4}$ where pillar score = $\frac{\sum Sub-pillar\ scores}{2}$ and sub-pillar scores is $\sum w_i.Indicators$. Here, $w_i^{'}s$ are derived from Principal Component Analysis (PCA). However, PCA assigns more weights to variables with larger variances and inappropriate to measure business climate by OECD and the index of environmental sustainability [9]. PCA weights were different for data accumulated for 11-years and averaging year-wise data [10].

Other scales are there to assess QOL and physical and mental health of old-aged people suffering from various diseases. However, a disease may be associated with other physical and mental diseases. For example, Multiple sclerosis may affect cardiovascular functions and causes brainstem lesions affecting autonomic pathways in the medulla, overall plaque burden, etc. [11].

Various scales used to measure factors associated with QOL suffer from methodological limitations and are not comparable due to different dimensions covered; different item formats, scoring methods, method of combining the chosen indicators/dimensions, without ensuring normally distributed scores needed for parametric statistical inferences.

Methodological limitations of scoring of different scales for measuring QOL and associated factors are:

- Ordinal scores emerging from K-point items, K= 2, 3, 4, 5… fail to satisfy equidistant property due to unequal and unknown distance between levels [12] and thus, meaningful addition of item/dimension scores is not possible [13]. Need to consider response-categories along with format of the questionnaire were suggested [14].

- Equal importance assigned to the items and dimensions despite different item-total correlations, different factor loadings, etc. assumes perfect substitutability among the items/dimensions i.e. poor score of a dimension can be compensated by surplus in another dimension, implying loss of information about multidimensional nature of QOL [15]. However, life expectancy cannot be compensated by income.

- Directions of the scales may be different. While low score in Mini-Mental State Examination (MMSE) [16], Montreal Cognitive Assessment (MoCA) [17], indicate severity, the reverse is true for Activities of Daily Living (ADL) [18]. Scales must ensure uniform direction by reverse scoring or by subtracting observed score from maximum possible scale score like ADL to support strong negative association between dementia and QOL.

- Different score-ranges of item scores (X) (like McGill Pain Questionnaire (MPQ), SF-36, etc.) are often normalized for uniform score-range using Min-Max transformation $Z = \frac{X - Min_X}{Max(X_i) - Min(X_i)} * 100$ where $0 \leq Z \leq 100$ shows relative performance instead of absolute performance. A change in Min_X can change rankings due to change in marginal rates of substitution [19]. For variable measured in ratio scale, the fixed zero-point gets altered by such transformation. If X is in percentage, $Max(X_i)$-$Min(X_i)$ is not meaningful. Accordingly, Human Poverty Index (HPI) considers 3rd root for HPI-1 and 4th root for HPI-2 of average of figures in percentage [20]. Different methods of normalization result in change in shape of distributions in different fashions and may influence the final scores.

- Different number of tasks in neurological tests (NTs) gives different contributions of sub-sections to total score. For example, in MMSE, out of total score of 30, 10 points are given to orientation, against only 1 point for constructional apraxia. Similarly, in MoCA, Visuospatial/Executive section has 6 points but 3 points for the Naming section. Orientation in MoCA, with 6-points, contributes more to total test score.
- Indicators in percentages (like percentages of older persons), ratios (like DR_{Oldage}, Plasma amyloid beta (Aβ)1-42/Aβ1-40 ratio), etc. are not additive. Combining ordinal scores generated by questionnaires; count data like family size, number of errors (Seashore Rhythm Test of HRB), biomarkers like number of plaques and tangles; ratio scale data like remaining life expectancy, time taken to complete tasks (Tactual Performance Test of HRB), etc. have inherent problems.

- QOL studies by weighted sum were criticized for limited statistical power [21] and were disfavoured [22].

- Reliability of WHOQOL-BREF exceeded reliability of SF-36 for people with schizophrenia [23].

For chosen n-indicators, the paper transforms ordinal item scores to normally distributed scores (P-scores) for meaningful arithmetic aggregation leading to scale score $S = \sum_i P_i$ reflecting overall QOL-status (QOL_{Total}) following normal distribution. A composite index of QOL of elderly persons at t-th time period ($I_{QOL-elderly_t}$) is also proposed by multiplicative aggregation of ratios of S_i's at t-th period and base period as $I_{QOL-elderly_t} = \frac{S_{1t}.S_{2t}..........S_{nt}}{S_{10}.S_{20}.....S_{n0}} * 100$. Both QOL_{Total} and $I_{QOL-elderly_t}$ satisfy desired properties including quantification of progress made by a country/region over time and drawing path of progress/decline since the base period.

4. Literature Survey

Scales to assess purpose in life involve different sub-scales and different number of items with different number of response-categories (levels). For example, subscales of Sense of Purpose in Life (SPIL) are awareness of purpose, awakening to purpose, and altruistic purpose [24], Life Attitude Profile (LAP) has seven subscales and 46 number of 7-point items [25], Psychological Well-Being (PWB) [26] consisting of 6-point items focuses primarily on general sense of intentionality and future-directedness, lacks multidimensional nature, comprehensive definition of purpose [27]. PWB fails to measure changes across time [28]. Meaning in Life Questionnaire (MLQ) with five items (7-point) in each of two subscales: presence of meaning (MLQ-P) and search for meaning (MLQ-S) assesses sense of meaning and purpose in life [29]. However, negatively worded 9th item of MLQ-P requiring reverse scoring is problematic and its removal was suggested [30]. Correlation between MLQ-S and MLQ-P among elderly gave mixed results. While [31] found $r_{MLQS,MLQP} = 0.40$ for Chinese elderly, the same was reported as -0.44 by [32] for Australian population, indicating interactions of cultural, contextual factors, social security, etc. may influence $r_{MLQS,MLQP}$ differently.

Cognitive impairment, progressive memory loss, low speed of information processing, significant disability, etc. are common for aged people suffering from neurodegenerative diseases like dementia [33]. Important pathological markers of Alzheimer's Disease (AD) like count of total number of plaques (diffuse or neurotic), dense neurofibrillary tangles in tissue samples from several brain regions may be skewed toward the lower end of values. Mixed evidences were observed regarding effect of cognitive impairment among people with dementia (PWD) and QOL [34] due to various non-overlapping dimensions of multidimensional QOLs and NTs and methodological limitations.

Illustrative list of specific scales to assess disabilities and psychological factors using Patient-Reported Scales (PRS) like Depression scale, Modified Somatic Perception Questionnaire (MSPQ) to study effect of depression and somatic anxiety, World Health Organization Disability Assessment Schedule (WHODAS), etc. are often skewed, with floor or ceiling effects, without ensuring normally distributed scores, needed for parametric statistical inferences [35,36]. Multidimensional aspects of pain are Sensory (Intensity, location, character of the pain sensation), Affective (Emotional and perceived components) and Impact (Disability or dysfunctions) - all affecting QOL. Based on changes in before and after surgery by SF-12, [37] found that major surgery decreases postoperative PCS-scores.

Empirical investigations gave contrasting results for socio-demographic variable. While WHOQOL-BREF produced no significant gender effect [38], results obtained by [39,40] with SF-36and [41] using WHOQOL-BREF showed $\overline{QOL}_{Male} > \overline{QOL}_{Female}$. For two items X and Y, interpretation and operations on X±Y are problematic when X and Y follow different distributions. Meaningfulness of X+Y = Z demands similar distribution of X and Y facilitating derivation of distribution of Z and enabling computation of $P(Z=z) = P(X=x, Y=z-x)$ for discrete case and $P(Z \leq z) = P(X + Y \leq z) = \int_{-\infty}^{\infty} (\int_{-\infty}^{z} f_{X,Y}(x, t-x) dt)\, dx$ for continuous case, and finding joint distribution of Z as sum of item/dimension scores. Satyendra NC. [42] defined $SF36_{Total}$ as sum of normally distributed item scores.

PCA of SF-36 showed inverse relationship between Physical Component Summary (PCS) and mental component summary (MCS) [43] implying good physical health pre-supposes poor mental health and vice versa. While negative association between retirement and SPIL was reported [44], inter-individual variability affected SPIL differently among the retires [45]. However, such associations may fail to give causal impact of retirement on SPIL. Correlation between dimensions of PWB ranged from 0.13 (Purpose in life & Autonomy), to 0.46 (Self-acceptance & Environmental mastery) and dimension reliability (Cronbach alpha) ranged from 0.33 (for Purpose in life) to 0.52 (for Self-acceptance) [46]. However, PWB scale focuses on positive functioning ignoring positive feelings.

Indicators of well-being contain positive feelings (e.g. happiness, satisfaction) and positive functioning (e.g. competence, engagement, self-acceptance) or combination of both.

10-item Well-being scale by [47] includes indicators of both positive feeling and positive functioning. But, different items are measured with different rating scales with different scoring approaches viz. low → high for four items measuring happiness, emotional stability, vitality and resilience; and high → low for the other six items measuring optimism, engagement, competence, meaning, positive relationships and self-esteem. Against two-factor structure emerged from Factor Analysis (FA); single general factor was found to fit the model better [48]. The chosen QOL-outcome measurements may focus on aspects of well-being and not on evaluation of life.

Chronic pain is common among the elderly population. A guide for chronic pain in the elderly developed by the American Society of Geriatrics favours use of multidimensional pain quantification tools for the elderly [49].The questionnaire Geriatric Pain Measure (GPM) contains 24- number of Yes-No type items for assessing pain intensity (7-items), pain at ambulation (2-items), pain at vigorous activities (3-items), pain during other activities (5-items) and disengagement (7-items) [50]. Validity of multidimensional GPM was given as correlation with multidimensional MPQ covering 4-dimensions: Sensory-discriminating, motivational-affective and cognitive-evaluative and miscellaneous components of pain. Questions arise on selection of criterion scale with non-uniform dimensions and the obtained validity reflecting validity of which dimension. For multidimensional scale, Factorial Validity (FV) defined as ratio of the first eigen value to the sum of all eigen values reflecting validity of the main factor for which the scale was developed was preferred [51]. FV can be computed from single administration of a scale avoiding the problems of selection of criterion scale.

Psychometric properties of multidimensional scales are routinely computed ignoring definition of reliability or verifying assumptions of Cronbach alpha like unidimensionality, same true score variances for all items and same relationship to the measured construct (equal factor loadings). However, alpha has been reported despite several independent factors emerged from PCA or FA. For example, against two-factor solution (memory factor and visuo-spatial factor) for Repeatable Battery for Assessment of Neuropsychological Status (RBANS) with 12 sub-tests, Cronbach alpha = 0.92 was found [52]. Battery reliability is ≠ Average of sub-tests reliabilities. Wechsler adult intelligence scale–Fourth Edition (WAIS-IV) had alpha = 0.98 against reliability of constituent scales ranging from 0.90 to 0.96 [53]. Avoiding unidimensionality assumption, [54] proposed theoretically defined reliability ($r_{tt\text{-}Theoritical}$) by dichotomizing a test into parallel subtests (g-th and h-th) and computing

$$r_{tt-Theoritical} = \frac{S_T^2}{S_X^2} = 1 - \frac{S_E^2}{S_X^2} = 1 - \frac{\frac{1}{N}\left[\|X_g\|^2 + \|X_h\|^2 - 2\|X_g\|\|X_h\| Cos\,\theta_{gh}\right]}{NS_X^2}$$

where N: Sample size; $\|X_g\| = \sqrt{\sum_{i=1}^{N} X_{ig}^2}$ is length of the g-th vector, $\|X_h\| = \sqrt{\sum_{i=1}^{N} X_{ih}^2}$ and θ_{gh} is the angle between the g-th and h-th vectors.

Reliability of a battery consisting of K-subscales (without weights) was derived in terms of sub-test reliabilities by

$$r_{tt(Battery)} = \frac{\sum_{i=1}^{K} r_{tt_i} S_{X_i}^2 + \sum_{i=1\, i \neq j}^{K} \sum_{j-1}^{K} 2\, Cov(X_i, X_j)}{\sum_{i=1}^{K} S_{X_i}^2 + \sum_{i=1\, i \neq j}^{K} \sum_{j-1}^{K} 2\, Cov(X_i, X_j)}$$

Test validity gets lower if proportion of high performer is more in the sample [55]. Review of cognitive screening tests by [56] showed poor evidences of validity/reliability; sensitivity/specificity; factor structures which often fail to meet statistical standards.

5. Proposed Method

5.1. Pre-adjustment of Data

i) Ensure each item is positively related to the construct i.e. higher the item score, higher is QOL
ii) Assign 1, 2, 3, 4, 5, etc. to the response-categories of items avoiding zero.

Transform ordinal scores of each K-point item to continuous equidistant scores (E_i-scores) by selecting weights ensuring $5W_5 - 4W_4 = 4W_4 - 3W_3 = 3W_3 - 2W_2 = 2W_2 - W_1 = \text{Constant} > 0$ for $K = 5$ (say). One way of getting such weights using frequencies of levels in various steps given by [57] is described below.

i) For an item, find proportion $p_i = \frac{f_i}{n} \; \forall \; i = 1,2,3.4.5$ where f_i: Frequency of i-th level of the item; n denotes number of individuals answering the item. Here, $\sum_{i=1}^{5} p_i = 1$
ii) Find Cumulative Proportions (C_i) where $C_1 = p_1$, $C_2 = p_1 + p_2$, ……., $C_5 = 1$
iii) Find area under the standard Normal curve (A_i) where A_i = Area Upto C_i. Clearly, $\sum_{i=1}^{5} A_i > 1$
iv) Find Initial Weights $w_i = \frac{A_i}{\sum_{i=1}^{5} A_i}$ so that $\sum_{i=1}^{5} w_i = 1.$ Here, $w_j > w_{j-1}$ for $j = 2,3,4,5$
v) Find correction factor α by dividing the difference between Maximum area and the Minimum area by 3. The modified areas Δ_1, Δ_2, Δ_3, Δ_4 and Δ_5 can be determined by taking $\Delta_1 = A_1$, $\Delta_2 = \Delta_1 + \alpha$; $\Delta_3 = \Delta_2 + \alpha$; $\Delta_4 = \Delta_3 + \alpha$; $\Delta_5 = \Delta_4 + \alpha$
vi) Define final weights $W_j = \frac{\Delta_j}{\sum_{j=1}^{5} \Delta_j}$ satisfying $\sum_{j=1}^{5} W_j = 1.$

Item-wise E_i-scores can be standardized by

$Z = \frac{E-\bar{E}}{SD(E)} \sim N(0,1)$ and transformed to P_i-scores by $P_i = (100-1)\left[\frac{Z_i - MinZ_i}{MaxZ_i - MinZ_i}\right]+1$ so that $1 \le P_i \le 100$ and $P_i \sim N(\mu_i,\sigma_i)$. Dimension scores ($D_j$) is taken as sum of relevant P_i's and scale score $S = \sum_j D_j = \sum_i P_i$. Here $S \sim N\left(\sum_i \mu_i, \left[\sum \sigma_i^2 + 2\sum_{i\neq j} Cov\left(H_i, H_j\right)\right]\right)$ and enables undertaking of parametric statistical analysis.

Normally distributed scores for each QOL scale and associated factors like MLQ-S and MLQ-P, LAP, PWB, Cognitive impairment, Intensity and stages of various diseases, Depression, Disability Assessment, Satisfaction with life, Positive feeling and functioning, etc. can be added to get overall QOL scores (QOL_{Total}) also following normal distribution. However, count data and ratio scale data need not be transformed to E_i-scores and can be straight standardized to Z-scores $\sim N(0,1)$ followed by $P_i = (99)\left[\frac{Z_{ij} - Min_{Z_{ij}}}{Max_{Z_{ij}} - Min_{Z_{ij}}}\right]+1$ where $1 \le P_i \le 100$. Empirical relationship can be established between QOL_{Total} and chosen factors to predict the former.

In addition, proposed scores of each associated factors, measure of QOL may be computed for t-th period and separately for the base period and their ratios can be combined by multiplicative aggregation to find composites index $I_{QOL-elderly_t} = \frac{S_{1t}.S_{2t}.........S_{nt}}{S_{10}.S_{20}.....S_{n0}}$ ignoring the n-th root of geometric mean of the ratios. $I_{QOL-elderly_t}$ reflects overall improvement/decline in the t-th period from the base period. The index may be computed separately for socio-economic-demographic factors to see behavior of the index over various age-related risk factors.

5.2. Properties

Each unit-free index QOL_{Total} and $I_{QOL-elderly_t}$ reflects QOL-status by continuous monotonic variable avoiding scaling, selection of weights, reducing substitutability among the component indicators. Satisfaction of equidistant property and normality of QOL_{Total} provides following advantages:

i) Meaningful "addition" to compute sample mean and variance and to estimate population mean, variance, and testing statistical hypothesis $H_0: \mu_{QOLtotal_t} = \mu_{QOLtotal_{t-1}}$ or $H_0: \mu_{QOLtotal-GrA} = \mu_{QOLtotal-GrB}$ using t-statistic and simultaneous testing of several means across demographic variables like gender, age, etc. by AVOVA.
ii) Better ranking and classification of old-aged persons.
iii) Avoids effect of outliers and produce no bias for developed or under-developed regions/countries.
iv) Integration of i-th and j-th scales using equivalent scores (x_{i0}, y_{j0}) by solving $\int_{-\infty}^{x_{i0}} f(x)dx = \int_{-\infty}^{y_{j0}} g(y)dy$ using standard normal table since S_i' follow normal [58]. Perfectly correlated equivalent cut-off scores give same results of ROC curve analysis for diagnosis by two scales.
v) Dimensions of QOL_{Total} can be ranked by relative importance given by $\frac{\Delta QOL_{Total}}{\Delta S_i}$ or by $\frac{\delta QOL_{Total}}{\delta S_i}$. Alternately, β-coefficients of regression of QOL_{Total} on $S_1, S_2, \ldots\ldots S_n$ may reflect relative importance.
vi) Progress/deterioration in successive years can be assessed by $\frac{QOL_t - QOL_{(t-1)}}{QOL_{(t-1)}} \times 100$ or by $\frac{I_{QOL-elderly_t} - I_{QOL-elderly_{(t-1)}}}{I_{QOL-elderly_{(t-1)}}} \times 100$. The ratio reflects responsiveness of the scale and also effectiveness of interventions/treatment plans. Decline in t-th period over (t-1)-th period for a sample requires identification of critical scale for which $\frac{S_{it}}{S_{i(t-1)}} < 1$ and decide corrective action plan in the identified direction.
vii) $I_{QOL-elderly_{t0}} * I_{QOL-elderly_{0t}} = 1$ implies satisfaction of time-reversal test. Similarly, $I_{QOL-elderly_{20}} = I_{QOL-elderly_{21}} * I_{QOL-elderly_{10}}$ indicates admissibility of formation of chain indices. These help to plot progress-path of a country across time since the base period and inter-regional comparisons.
viii) The two indices are highly correlated since $log\, I_{QOL-elderly_t} = log \sum_{i=1}^{n} S_{it} - log \sum_{i=1}^{n} S_{i0} = log\left(QOL_{Total}\right) - log \sum_{i=1}^{n} S_{i0}$.
ix) Moreover, $log \sqrt[n]{I_{QOL-elderly_t}} = \frac{1}{n}\sum_{i=1}^{n} log\, Y_i$ where $Y_i = \frac{S_{it}}{S_{i0}}$. Thus, Geometric Standard Deviation (GSD) (SD_{GM}) can be derived as log $SD_{GM} = [\frac{1}{n}\sum_{i=1}^{n}\left(log\, Y_i - log\, GM\right)^2]^{\frac{1}{2}}$ $\Rightarrow$ log (GSD of Y_1, Y_2, … …, Y_n) = usual SD of log Y_1, log Y_2, ………, log Y_n

For large sample, population estimate of GM is sample GM and estimate of standard error of the GM is $GM.\left(\frac{\text{Log } S_{GM}}{\sqrt{n-1}}\right)$ [59]. Thus, $H_0: \sqrt[n]{I_{QOL-elderly_{ti}}} = \sqrt[n]{I_{QOL-elderly_{tj}}}$ for countries $i \neq j$ or $H_0: \sqrt[n]{I_{QOL-elderly_{tj}}} = \sqrt[n]{I_{QOL-elderly_{(t-1)j}}}$ for the j-th country can be tested by t-tests using logarithms of the observations.

i. Factorial validity (FV) = $\frac{\text{highest eigen value}\left(\lambda_1\right)}{\sum \lambda_i}$ is related to test variance S_X^2 for standardized item scores ($FV_{Z\text{-}scores}$) as

$$S_X^2 = \sum \lambda_i \; S_X^2 = \sum \lambda_i + 2\sum_{i\neq j=1}^{m} Cov\left(X_i, X_j\right) = \frac{\lambda_1}{FV} + 2\sum_{i\neq j=1}^{m} Cov\left(X_i, X_j\right) \Rightarrow r_{tt(theoretical)} = \frac{S_T^2}{\frac{\lambda_1}{FV} + 2\sum_{i\neq j=1}^{m} Cov\left(X_i, X_j\right)}$$

for a test with m-items.

Maximum reliability of a test (α_{PCA}) = $\left(\frac{m}{m-1}\right)\left(1 - \frac{1}{\lambda_i}\right)$ [60] is related to FV by

$$\alpha_{PCA} = \left(\frac{m}{m-1}\right)\left(1-\frac{1}{\lambda_i}\right) = \left(\frac{m}{m-1}\right)\left(1-\frac{1}{FV.\sum\lambda_i}\right) = \left(\frac{m}{m-1}\right)\left(1-\frac{1}{m.FV_{Z-scores}}\right)$$

indicating higher $FV_{Z\text{-}scores}$ increases α_{PCA}.

6. Discussions

Each proposed index avoids equal importance to items and generates continuous, monotonic scores. Normally distributed QOL_{Total} and $\log \sqrt[n]{I_{QOL-elderly_t}}$ satisfy desired properties including estimation of first and second central moments of each index for a group of regions say states or countries at a given time period. However, increase in number of dimension or indicators will increase value of the index, which can be easily mitigated by $\frac{QOL_{Total}}{Number\ of\ indicators\ (n)}$ and $\sqrt[n]{I_{QOL-elderly_t}}$

Dimension-wise indices can be constructed considering indicators relevant to a dimension ensuring aggregation of dimensions = aggregation of component indicators. $I_{QOL-elderly_t}$ is in variant under change of scale and can consider all indicators in percentages or in ordinal scale or skewed.

Equivalent cut-off scores of class boundaries of two scales can be found satisfying $\frac{Var.of\ group_{score \geq S_0 For\ Scale\ 1}}{Variance\ of\ S_1} = \frac{Var.of\ group_{score \geq S_0 For\ Scale\ 2}}{Variance\ of\ S_2}$ facilitating similar classification– efficiency in terms of within group variance and between group variance.

Empirical relationship of $\boldsymbol{QOL_{Total}}$ on the chosen scales can be found keeping in mind:

R^2 for $QOL_{Total} = \alpha_1 + \sum_{i=1}^{k} \beta_i S_i$ exceeds R^2 for $QOL_{Total} = \alpha_2 + \beta(S_1 + S_2 + \ldots\ldots\ldots + S_k)$ even if the same items used in both regression models [61]. The contradictory result could be due to reduction of number of independent variables in the second model and existence of bad leverage points for regression equation $Y = \alpha + \beta X$. If r_i is an outlier in the set of residuals $\{r_1, r_2, \ldots \ldots, r_n\}$ and the corresponding (X_i, Y_i) is a leverage point, then (X_i, Y_i) is a bad leverage point implying poor fit of the linear model. Different approaches are there for detecting bad leverage points of multiple linear regression equations. Method of computing slope and intercept of regression equation after removing bad leverage points was proposed [62].

Validity of a multidimensional scale from a single administration can be obtained as FV to reflect validity of the main factor for which the test was developed. However, for scales to assess physical and/or mental disorders, clinically meaningful content validity is required. Relationships derived between FV and maximum reliability of a test and also between FV and $r_{tt(theoretical)}$ can be used effectively to compare scales.

The proposed measures improve quality of measurements of scale and aggregation of scale scores facilitating meaningful comparisons across groups and time and are critically relevant to planners and researchers.

7. Conclusions

Proposed method of aggregations of count data, variables in ratio scale and in ordinal scale ensuring normal distribution of the proposed indices is an improvement of assessment of QOL for elderly persons with benefits of parametric analysis for meaningful analysis across time and space. Suggested integration of several scales has clear theoretical advantages. Assumption-free measures of reliability, validity, etc. may be used while comparing multidimensional QOL scales. Proposed indices QOL_{Total} by arithmetic aggregation and $I_{QOL-elderly_t}$ by multiplicative aggregation are highly correlated. From the angle of distribution, QOL_{Total} may be preferred over $I_{QOL-elderly_t}$. But, $I_{QOL-elderly_t}$ may be preferred for comparison across time because of satisfaction of time-reversal test, formation of chain indices, and ability to consider all chosen indicators and dimensions, etc.

Future action to improve QOL of elderly population could be effective family interventions based on cognitive behavioral therapy and implementation of robust support system at national level in terms of better social security for old-aged people including widows, creating opportunities to utilize skill and experience of elderly people, access to health care, empowerment and welfare with emphasis on the unorganized sector.

8. Declarations

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